Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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6249494486

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **May/June 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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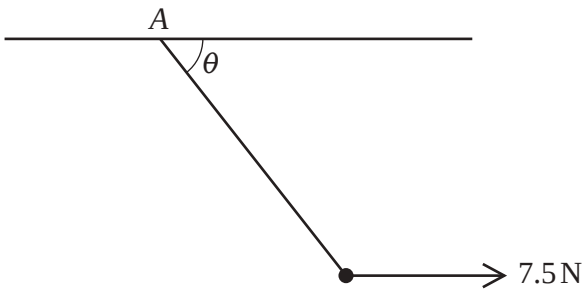

A particle of weight 10 N is attached to one end of a light elastic string. The other end of the string is attached to a fixed point A on a horizontal ceiling. A horizontal force of 7.5 N acts on the particle. In the equilibrium position, the string makes an angle θ with the ceiling (see diagram). The string has natural length 0.8 m and modulus of elasticity 50 N.

(a) Find the tension in the string. [2]

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(b) Find the vertical distance between the particle and the ceiling. [3]

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2 One end of a light inextensible string of length a is attached to a fixed point O. A particle of mass m is attached to the other end of the string. The particle is held at the point A with the string taut. The angle between OA and the downward vertical is equal to α, where $\cos\alpha = \frac{4}{5}$. The particle is projected from A, perpendicular to the string in an upwards direction, with a speed $\sqrt{3ga}$. It then moves along a circular path in a vertical plane. The string first goes slack when it makes an angle θ with the upward vertical through O.

Find the value of $\cos\theta$. [5]

3 A particle P is moving in a horizontal straight line. Initially P is at the point O on the line and is moving with velocity $25\,\text{m}\,\text{s}^{-1}$. At time t s after passing through O, the acceleration of P is $\frac{4000}{(5t+4)^3}\,\text{m}\,\text{s}^{-2}$ in the direction PO. The displacement of P from O at time t is x m.

Find an expression for x in terms of t. [5]

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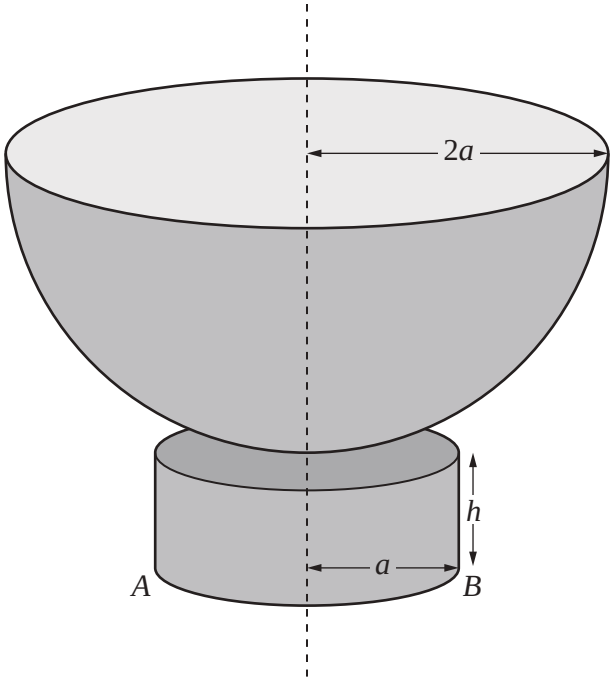

An object is composed of a hemispherical shell of radius $2a$ attached to a closed hollow circular cylinder of height h and base radius a. The hemispherical shell and the hollow cylinder are made of the same uniform material. The axes of symmetry of the shell and the cylinder coincide. AB is a diameter of the lower end of the cylinder (see diagram).

(a) Find, in terms of a and h, an expression for the distance of the centre of mass of the object from AB. [4]

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The object is placed on a rough plane which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{2}{3}$. The object is in equilibrium with AB in contact with the plane and lying along a line of greatest slope of the plane.

(b) Find the set of possible values of h, in terms of a. [4]

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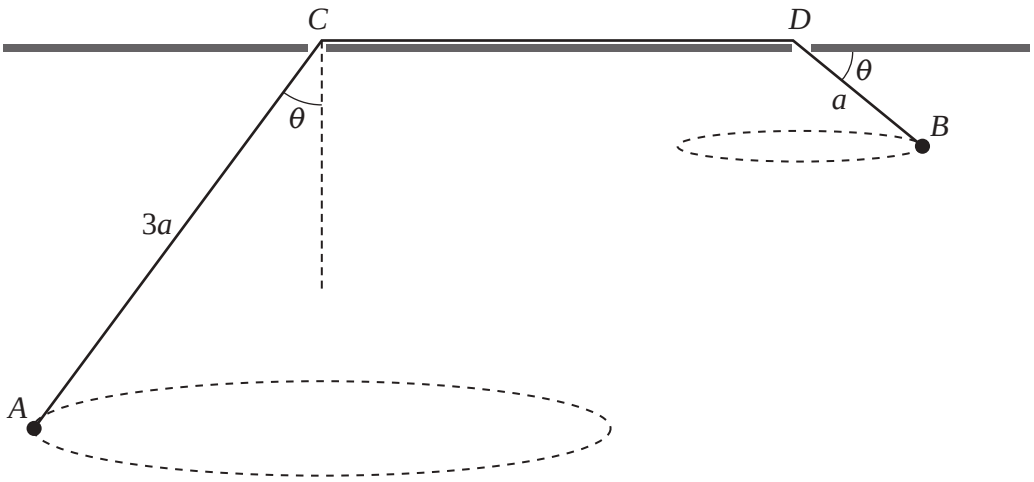

A light inextensible string AB passes through two small holes C and D in a smooth horizontal table where $AC = 3a$ and $DB = a$. A particle of mass m is attached at the end A and moves in a horizontal circle with angular velocity ω. A particle of mass $\frac{3}{4}m$ is attached to the end B and moves in a horizontal circle with angular velocity $k\omega$. AC makes an angle θ with the downward vertical and DB makes an angle θ with the horizontal (see diagram).

Find the value of k. [7]

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6 Two uniform smooth spheres A and B of equal radii have masses m and km respectively. The two spheres are on a horizontal surface. Sphere A is travelling with speed u towards sphere B which is at rest. The spheres collide. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a) Show that the speed of B after the collision is $\frac{3u\cos\alpha}{2(1+k)}$ and find also an expression for the speed of A along the line of centres after the collision, in terms of k, u and α. [4]

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After the collision, the kinetic energy of A is equal to the kinetic energy of B.

(b) Given that $\tan\alpha = \frac{2}{3}$, find the possible values of k. [5]

7 Particles P and Q are projected in the same vertical plane from a point O at the top of a cliff. The height of the cliff exceeds 50 m. Both particles move freely under gravity. Particle P is projected with speed $\frac{35}{2}\,\text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Particle Q is projected with speed $u\,\text{m s}^{-1}$ at an angle β above the horizontal, where $\tan\beta = \frac{1}{2}$. Particle Q is projected one second after the projection of particle P. The particles collide T s after the projection of particle Q.

(a) Write down expressions, in terms of T, for the horizontal displacements of P and Q from O when they collide and hence show that $4uT = 21\sqrt{5}(T+1)$. [4]

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(b) Find the value of T. [4]

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(c) Find the horizontal and vertical displacements of the particles from O when they collide. [3]

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Additional page

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